Introduction to evolutionary finance

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The problem: Efficient Market Hypothesis

Friedman or Alchian argument is twofold

- irrational traders perform badly and are driven from the market
- rational traders drive the prices to their fundamental values

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Overview

We investigate wealth-driven selection in incomplete asset markets populated by heterogeneous investors without perfect foresight assumption.

- Which investment rules (beliefs & preferences, ...) does the market reward?
- Does it exist a dominant rule?
- Can investment behaviors be “ordered”?
- Is agents’ heterogeneity a persistent property?
- What are the consequences for asset prices?
- Do asset prices reflect the most accurate beliefs?

We provide answers a simple, but rich enough (stochastic, behaviors/rules, equilibrium prices), analytically tractable model by studying the local stability of market selection equilibria.

The Model: Assets

Discrete time. S possible states of the world realized with fixed probability $(q_1, \ldots, q_S)$ at each t. Ω space of sequences $\omega = (\omega_1, \ldots, \omega_t, \ldots)$. Stationary and ergodic process.

Repeated exchange of K short-lived assets which represent contingent claims on future (uncertain) dividends. Asset k payoff at time t is $D_{k,s}$ if $\omega_t = s$. D is the dividend matrix, full rank (in particular no zero rows, no zero columns). $P_{k,t}$ is price of asset k at time t.

$$D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, D = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, D = \begin{pmatrix} 1 & 1 \\ 2 & \frac{1}{2} \end{pmatrix}, D = \begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 2 \end{pmatrix}$$

Prices and Wealth Dynamics

W_t^i is the wealth of agent i at time t. A fraction $\alpha_{k,t}^i$ is invested to buy $h_t^i = \alpha_{k,t}^i W_t^i / P_{k,t}$ shares in asset k while $1 - \sum_k \alpha_{k,t}^i$ is consumed. By Walrasian market clearing

$$1 = \sum_i h_{k.t}^i \Leftrightarrow P_{k,t} = \sum_i \alpha_{k,t}^i W_t^i \, .$$

Wealth at time $t+1$ depends on the realization of the state of the world ω_{t+1}

$$W_{t+1}^i = \sum_{k=i}^{K} \frac{\alpha_{k,t}^i W_t^i}{P_{k,t}} D_{k,\omega_{t+1}} \, .$$

Background Literature

Simple, “exogenous”, rules (wealth fractions depend only on dividend process)

- Blume and Easley (1992) *Evolution and Market Behavior*. Market for Arrow securities and simple rules. Selection rewards log preferences with beliefs ”closest” (relative entropy) to π. Relative entropy defines order relation. (in background Log-optimality: Kelly, 1956; Breiman, 1961).
- *Evolutionary Finance* (a survey Evstigneev, Hens, and Schenk-Hoppe’, 2009). Simple rule in an extended framework (long-lived assets, possibly incomplete markets, more general dividend and learning processes). G-Kelly rule, i.e. invest proportionally to expected dividends, is global maximum w.r.t. order relation.

Related Literature

Non-simple, “endogenous”, rules (wealth fractions depend on dividend and price process)

- Sandroni (2000) and Blume and Easley (2006): general demands, infinite horizon, perfect foresight on prices, dynamically complete markets. Find that Pareto optimality implies that, controlling for discount rates, best beliefs (relative entropy terms) gain all wealth in long run.
- Some finance applications of Heterogeneous Agents Models (Hommes 2006) and Agent Based Computational Economics (LeBaron 2006) study wealth-driven selection of CRRA rules in deterministic/simulation framework. Partial equilibrium framework. Levy, Levy, Solomon (1994, 1995, 2000), Chiarella et al (2001, 2006), Le Baron (2012). Behavioral Finance.

Our Framework

- Short-lived assets (K)
- Endogenous investment rules (I) (L)
- No perfect foresight (incompleteness)
- Repeated trade in discrete time, temporary equilibrium
- Random Dynamical System (I) $\times$ (K) $\times$ (L)
- Local (and global) stability analysis of long-run market selection equilibria

Today we discuss:

- Local stability analysis (hint global)
- Market selection landscape depends on the ecology of traders, no ordering
- Heterogeneity may be persistent (time varying)
- Asset prices may not reflect beliefs of best informed agent
- Never vanishing rule exists

Investment Rules

Generalized CRRA

Agent i invests on asset k at time t a fraction $\alpha^i_{k,t}$ of her wealth. We assume that, given a time-independent function of assets' prices, α^i_k, it holds

$$\alpha^i_{k,t} = \alpha^i_k(\mathbf{P}_t, \mathbf{P}_{t-1}, ...; D, \pi) \quad k = 1, \ldots, K\,, \tag{1}$$

where $\mathbf{P}_t$ is period t price vector (CRRA included, CARA excluded).

- **P1** Each agent i consumes in $[0, W^i)$, or $\sum_{k=1}^K \alpha^i_k(\mathbf{P}_t, ...) = \delta^i_t = 1 - \alpha^i_{0,t} \in (0, 1]$;
- **P2** Portfolios are maximally diversified, or $\sum_{k=1}^K \alpha^i_k(\mathbf{P}, ...)D_{k,s} > 0$ for every s and i.
- **P3** Demand is strictly positive for zero contemporaneous prices, that is, for every asset k and agent i, $\alpha^i_k(\mathbf{P}_t, ...)/P_{k,t} \to c > 0$ if $P_{k,t} \to 0$.

Normalized Prices and Wealth Dynamics

Normalizations leads to:

$$\sum_k d_{k,s} = 1, \quad \sum_i w_t^i = 1, \quad \sum_k p_{k,t} = \sum_i (1 - \alpha_{0,t}^i) w_t^i \quad \text{for all } t$$

Normalized inter-temporal budget constraint ω_{t+1}

$$w_{t+1}^i = \sum_k \frac{\alpha_k^i(\mathbf{p}_t, \ldots; D, \pi) d_{k,\omega_{t+1}}}{p_{k,t}} \, w_t^i \, .$$

Normalized Walrasian market clearing

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Market Dynamics as a Random Dynamical System

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(w_{t+1}, p_{t+1}) = \mathcal{F}(\omega_{t+1})(w_t, p_t) = \left[\begin{array}{lcl}
w^1_{t+1} & = & \mathcal{W}^1(w_t, p_t; \omega_{t+1}) \\
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w^I_{t+1} & = & \mathcal{W}^I(w_t, p_t; \omega_{t+1}) \\
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p_{K,t+1} & = & f_K(w_t, p_t; \omega_{t+1}) \\
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Dominance and Survival

Given (w_0, p_0) and given sequence ω we get trajectories, sequences of wealth fractions $\{w\}$ and prices $\{p\}$, and define

Definition

An agent i is said to survive on a given trajectory generated by the market dynamics if $\limsup_{t\to\infty} w_t^i > 0$ on this trajectory. Otherwise, an agent n is said to vanish on a given trajectory. A surviving agent i is said to dominate on a given trajectory if she is the unique survivor on that trajectory, that is, $\liminf_{t\to\infty} w_t^i = 1$

Survival and dominance can be characterized for subsets of Ω, e.g. almost surely.

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Long-run Market Selection Equilibria

We identify long-run market equilibria as states where w, p, αs are constant.

Technically these are deterministic fixed point of the random dynamical system.

Definition

Consider the stochastic process with elements $\omega \in \Omega$. The state (w^*, p^*) is a deterministic fixed point of the random dynamical system φ generated by the family of maps if, for almost all $\omega \in \Omega$, it holds

$$\varphi(t, \omega, w^*, p^*) = (w^*, p^*) \,, \quad \text{for every} \quad t \tag{2}$$

Survival and dominance at a market selection equilibrium

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Survival and dominance at a market selection equilibrium

Known Results: Simple Rules and Arrow Securities

Blume and Easley (1992) - 2 agents, 2 assets, no consumption

Wealth dynamics:

$$w^i_{t+1} = \begin{cases} \frac{\alpha^i w^i_t}{p_t} & \omega_{t+1} = 1 \\ \frac{(1-\alpha^i) w^i_t}{1-p_t} & \omega_{t+1} = 2 \end{cases},$$

where price (only one asset matters due to constant sum)

$$p_t = \alpha^1 \, w^1_t + \alpha^2 \, w^2_t .$$

Price is in between the αs.

Each period the market rewards the agent with a higher stake in the dividend paying asset.

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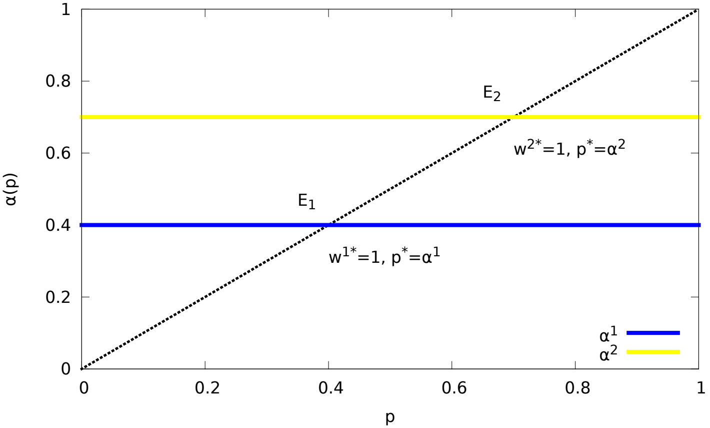
Simple Rules

Market equilibria in a plot



Simple Rules and Arrow Securities

Wealth ratio dynamics

When computing wealth ratios prices simplify away

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Define the Relative Entropy of α w.r.t. to π

$$I_\pi(\alpha^i) = \sum_s \pi_s \log \frac{\pi_s}{\alpha_s^i} \geq 0 \quad \text{then} \quad \frac{1}{T} \log \frac{w_T^1}{w_T^2} \to \left(I_\pi(\alpha^2) - I_\pi(\alpha^1)\right) .$$

If $I_\pi(\alpha^1) < I_\pi(\alpha^2)$ then $w_T^1 \to 1$ at exponential rate and agent 1 dominates globally.

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Simple Rules

The random walk view

Define

$$x_t = \log \frac{w_t^1}{w_t^2}$$

then the wealth dynamics is

$$x_{t+1} = x_t + \mu + \epsilon_{t+1}$$

where

$$\mu = I_\pi(\alpha^2) - I_\pi(\alpha^1)$$

and $\{\epsilon\}$ are i.i.d random variables with zero mean and finite variance.

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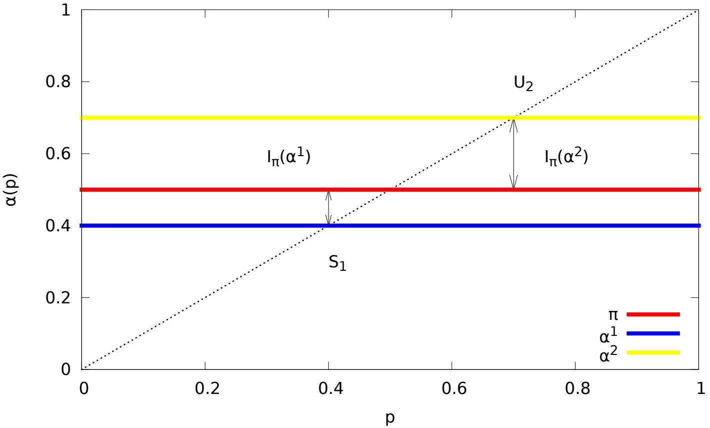
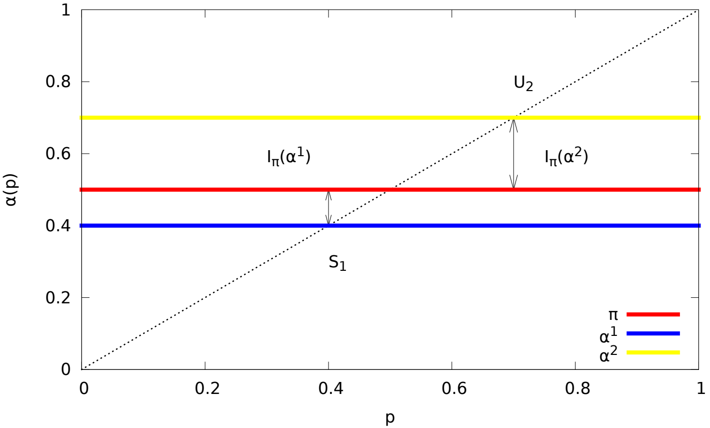


Two consequences:

- 1 No heterogeneity, best informed wins, rules ordered in survivability: $\alpha \succeq \beta$
- 2 The Kelly rule $\alpha_k = \pi_k$ dominates on Ω, $I(\text{Kelly}) = 0$.

Simple Rules

Wealth selection in a plot



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Non-simple Investment Rules

Market dynamics

Wealth dynamics is still:

$$w_{t+1}^i = \begin{cases} \frac{\alpha^i(p_t)w_t^i}{p_t} & \omega_{t+1} = 1 \\ \frac{(1-\alpha^i(p_t))w_t^i}{1-p_t} & \omega_{t+1} = 2 \end{cases}, \quad (3)$$

where $p_t(w_t)$ is the implicit solution of

$$p_t = \alpha^1(p_t)w_t^1 + \alpha^2(p_t)w_t^2 \,. \quad (4)$$

(if $w^{1*}\partial_p\alpha^1(p^*) + w^{2*}\partial_p\alpha^2(p^*) \neq 1$ OK around (w^*, p^*))

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Market Selection Equilibria

2 assets, 2 agents

Theorem

Market Selection Equilibria, that is, fixed points of the random dynamical system that corresponds to the market dynamics, are given by

$$
\begin{aligned}
w^* &= (1,0) \\
w^* &= (0,1)\,,
\end{aligned}
$$

which corresponds to single survivor equilibria of $i = 1, 2$ respectively and where $p^* = \alpha^i(p^*)$, or

$$w^* = (w^{1*}, 1 - w^{1*}) \quad w^{1*} \in (0,1)$$

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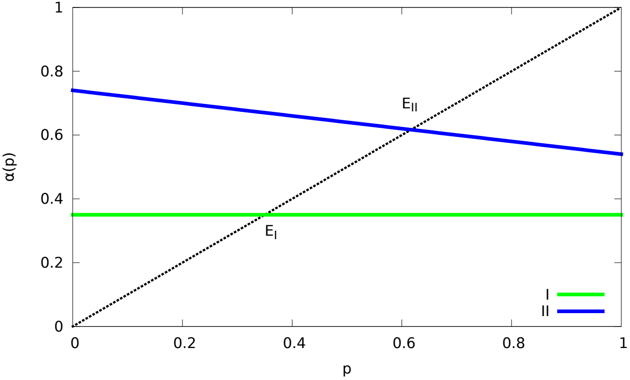
2 agents, 2 assets, non-simple investment rules

Market equilibria in a plot

$$p_t = \alpha^1(p_t)\, w_t + \alpha^2(p_t)(1 - w_t)$$



Non-simple Investment Rules

Selection

Overall we can compute

$$\frac{w^1_{t+1}}{w^2_{t+1}} = \begin{cases} \frac{\alpha^1(p_t)}{\alpha^2(p_t)} \frac{w^1_t}{w^2_t} & \omega_{t+1} = 1 \\ \frac{1-\alpha^1(p_t)}{1-\alpha^2(p_t)} \frac{w^1_t}{w^2_t} & \omega_{t+1} = 2 \end{cases}$$

Now, in T periods the ratio $\frac{w^1_T}{w^2_T}$ depends on the price history.

Non-simple Investment Rules

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Non-simple Investment Rules

The non-homogeneous random walk view

Define

$$x_t = \log \frac{w_t^1}{w_t^2}$$

then wealth dynamics gives

$$x_{t+1} = x_t + \mu(x_t) + \epsilon_{t+1}(x_t)$$

where

$$\mu(x_t) = I_\pi(\alpha^2(x_t)) - I_\pi(\alpha^1(x_t))$$

and $\{\epsilon\}$ are independent but non identically distributed random variables with zero mean and finite variance. Note that μ and its sign are now state dependent.

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Local Stability of Market Selection Equilibria

Definition

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A deterministic fixed point (w^*, p^*) of the random dynamical system $\varphi(t, \omega, w, p)$ is called asymptotically stable if, for almost all $\omega \in \Omega$ and there exists $U(\omega)$ of (w^*, p^*) such that for all (w, p) in $U(\omega)$ $\lim_{t\to\infty} ||\varphi(t, \omega, w, p) - (w^*, p^*)|| \to 0$.

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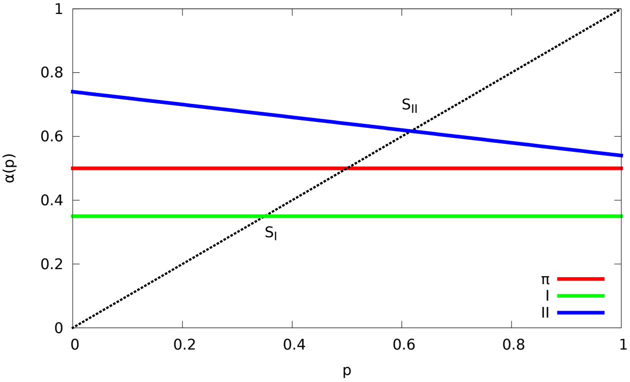
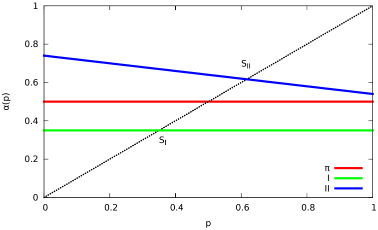
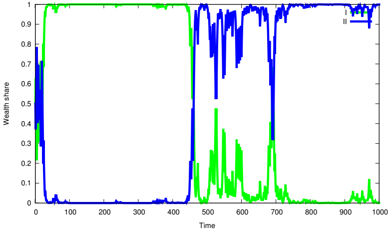
2 agents, 2 assets, non-simple rules

Stability in a plot



Coexistence of Stable (long-run) Equilibria





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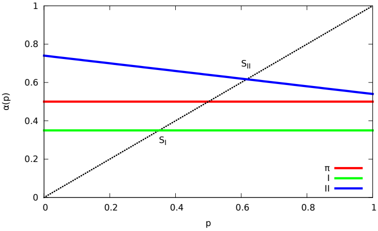

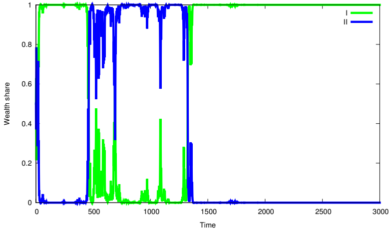

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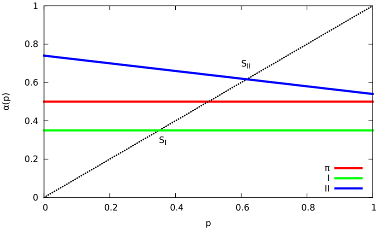

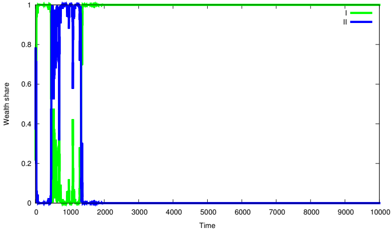

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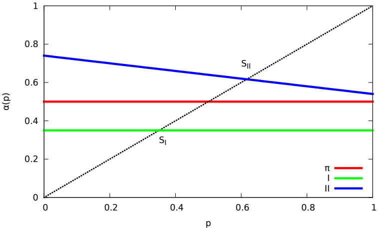

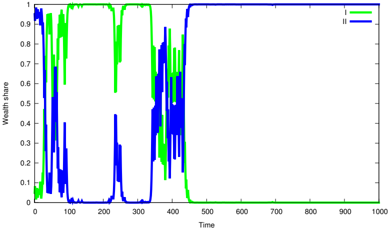

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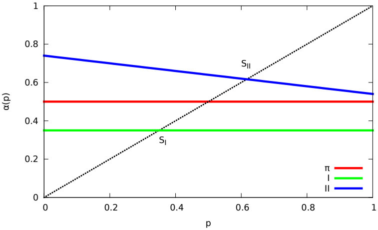

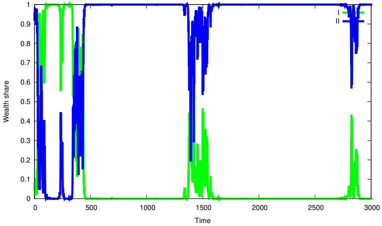

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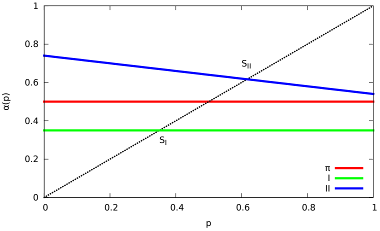
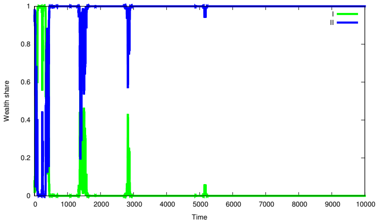
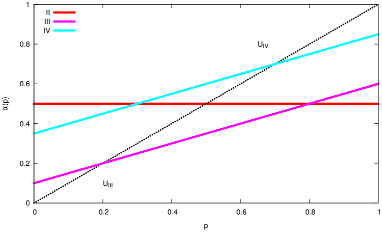
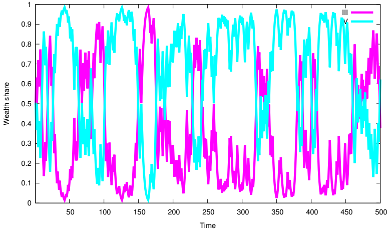




Multiple Unstable (long-run) Equilibria





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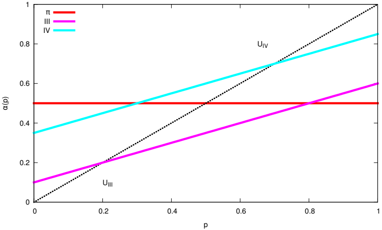
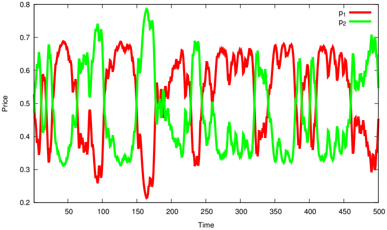
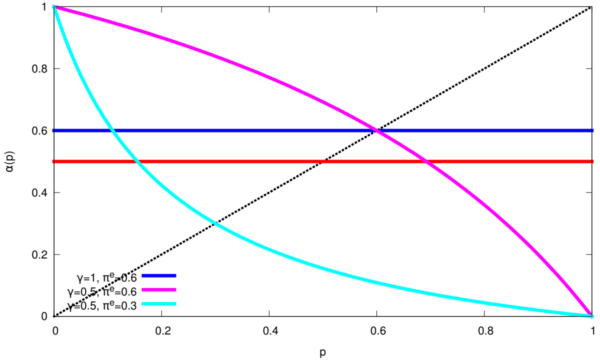




CRRA and no Aggregate Risk



$U = \sum_{\omega^T \in \Omega^T} \pi^e(\omega^T) u(w_T)$, with $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

Max U is solved by $\alpha(p; \pi^e, \gamma) = \dfrac{\left(\pi^e / p^{1-\gamma}\right)^{\frac{1}{\gamma}}}{\left(\pi^e / p^{1-\gamma}\right)^{\frac{1}{\gamma}} + \left((1-\pi^e)/(1-p)^{1-\gamma}\right)^{\frac{1}{\gamma}}}$

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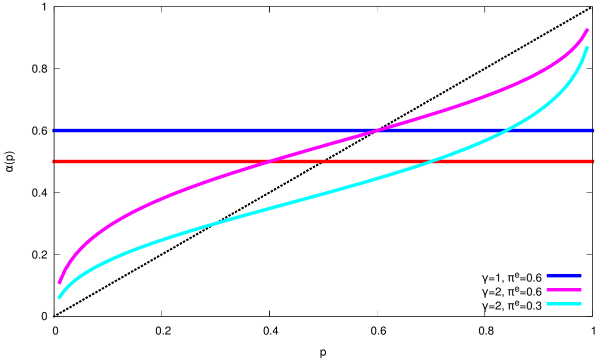

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An Order Relation on Rules

Given an asset market and two rules α and β define

$$\alpha \succeq \beta$$

iff α almost never vanishes when trading with β, and

$$\alpha \succ \beta$$

iff α dominates β.

Simple rules: complete, transitive
Non-simple rules: non-complete, non-transitive

Problems with Ordering

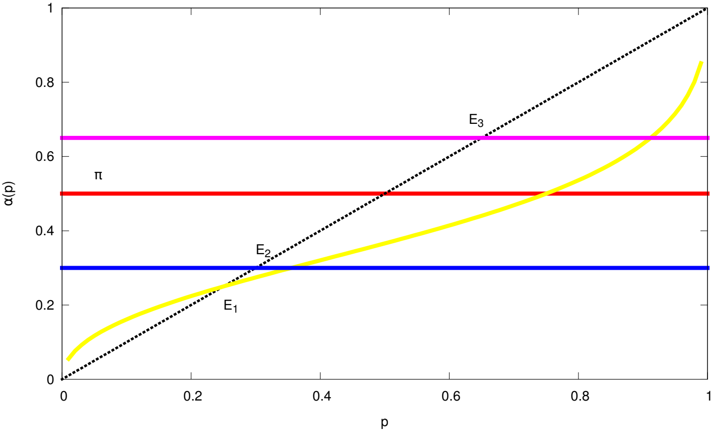
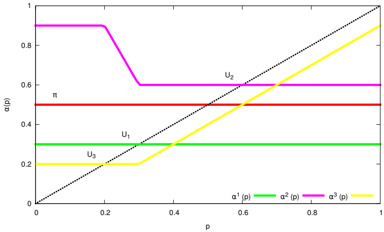
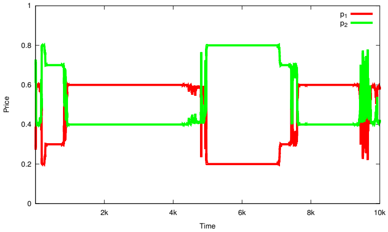


Figure: $\alpha^3 \succ \alpha^2$, $\alpha^2 \succ \alpha^1$, $\alpha^3 \sim \alpha^1$.

General Equilibrium Model of bubble and crashes

... in just one plot!





Learning from Prices

Rules αs can contain past price dependence because:

- Agents need to form price expectations when optimize over more periods
- Agents may want to use technical rules to exploit asset market imperfections
- Agents may want to use past price statistics to estimate fundamentals

Learning from prices means past prices feed-back into investment decisions

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Local Stability when Learning

Theorem

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If the eigenvalues of the infinitely iterated map are inside the unit circle then the deterministic fixed point is asymptotically stable. For fixed points of the type $(w^* = (1,0), \alpha^1(p^*) = p^*, p^*)$ *eigenvalues are*

$$\mu = \left(\frac{\alpha^2(p^*)}{\alpha^1(p^*)}\right)^{\pi} \left(\frac{1-\alpha^2(p^*)}{1-\alpha^1(p^*)}\right)^{1-\pi} \quad \text{and} \quad \lambda = \left.\frac{\partial \alpha^1(p)}{\partial p}\right|_{p^*}$$

For fixed points of the type $(w^*, \alpha^1(p^*) = \alpha^2(p^*) = p^*, p^*)$ *instead (stability)*

$$\mu = 1 \quad \text{and} \quad \lambda = w^* \left.\frac{\partial \alpha^1(p)}{\partial p}\right|_{p^*} + (1-w^*) \left.\frac{\partial \alpha^2(p)}{\partial p}\right|_{p^*}$$

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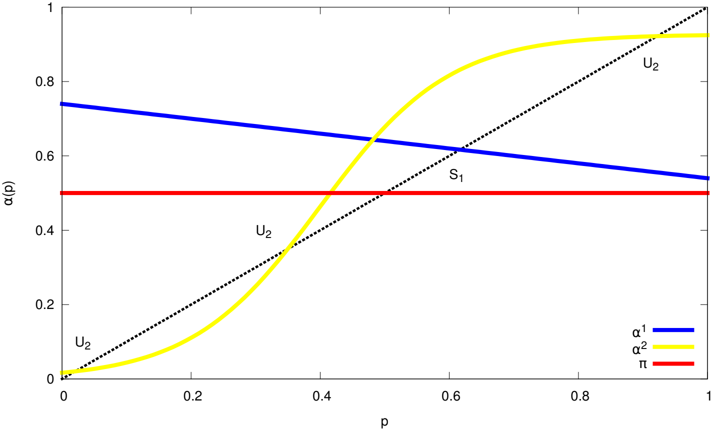
Local Stability in a Plot



Generalizations (see Bottazzi and Dindo JEE and WP)

1. K assets, I agents, L lags
2. Ergodic and stationary process rules states of the world. Entropy w.r.t. invariant measure matters
3. Local stability for both single and multiple survival
4. Global dominating rule (generalized Kelly)

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Beyond Toy Market

Local stability single survival

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Consider the fixed point $x^* = (w^*, p^*)$ where $w^{l*} = 1$ and $p^*_k = \alpha^l_k(p^*)$ for every $k = 1, \ldots, K$. Eigenvalues are

$$\mu_i = \prod_{s=1}^{K} \left(\sum_{k=1}^{K} \frac{\alpha^i_k(p^*)}{\alpha^l_k(p^*)} d_{s,k} \right)^{\pi_s}, \quad i \in 1, \ldots, I-1 \ ,$$

and solutions of the polynomial in λ of LKth degree

$$P(\lambda) = \sum_{l_1=1}^{L} \ldots \sum_{l_K=1}^{L} \lambda^{LK - \sum_j l_j} \sum_{\sigma} sgn(\sigma) \prod_{k=1}^{K} \left((\Delta^l_k)^{\sigma_k, l_{\sigma_k}} - \lambda\, \delta_{k,\sigma_k}\, \delta_{l_{\sigma_k},1} \ , \right)$$

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$$(\Delta_k^l)^{h,l} := -\sum_{k'=1}^{K} \{H^{-1}\}_{k,k'} (\alpha_{k'}^l)^{h,l} \ ,$$

and

$$H := \begin{pmatrix} (\alpha_1^l)^{1,0} - 1 & (\alpha_1^l)^{2,0} & (\alpha_1^l)^{3,0} & \dots & (\alpha_1^l)^{K,0} \\ (\alpha_2^l)^{1,0} & (\alpha_2^l)^{2,0} - 1 & (\alpha_2^l)^{3,0} & \dots & (\alpha_3^l)^{K,0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (\alpha_K^l)^{1,0} & (\alpha_K^l)^{2,0} & (\alpha_K^l)^{3,0} & \dots & (\alpha_K^l)^{K,0} - 1 \end{pmatrix} ,$$

non-singular, with

$$(\alpha_k^i)^{h,l} := \left. \frac{\partial \alpha_k^i(p)}{\partial p_h^l} \right|_{x^*} \ , \quad i = 1, \dots, I \ , \quad l = 0, 1, \dots, L \ , \quad k, h = 1, \dots ,$$

The dominant rule

A price dependent generalization of the Kelly rule

Define the function

$$I_\pi(\alpha, \mathbf{p}) = -\sum_{s=1}^{S} \pi_s \log \left(\sum_{k=1}^{K} \frac{\alpha_k}{p_k} d_{s,k} \right) ,$$

where d is the normalized dividend payoff matrix and π the invariant measure.

We define α^S as

$$\alpha^S(\mathbf{p}) = \mathrm{argmin}_{\alpha \in \Delta_c^K} \{\exp I_\pi(\alpha, \mathbf{p})\} . \quad (5)$$

If $D = I$, Arrow securities, then $\alpha^S = \pi$.

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If $D = I$, Arrow securities, then $\alpha^S = \pi$.

The dominant rule

A price dependent generalization of the Kelly rule

Define the function

$$I_\pi(\alpha, \mathbf{p}) = -\sum_{s=1}^{S} \pi_s \log \left(\sum_{k=1}^{K} \frac{\alpha_k}{p_k} d_{s,k} \right) ,$$

where d is the normalized dividend payoff matrix and π the invariant measure.

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Evolutionary stability and α^S

Theorem

Consider an ecology $\mathcal{E}$ of rules with $\alpha^S \in \mathcal{E}$. All deterministic fixed points $x^* = (w^*, p^*)$ where α^S vanishes are unstable. Moreover, there exists at least one stable deterministic fixed point in which α^S survives and long-run asset prices are equal to $p_k^* = \sum_{s=1}^{S} \pi_s d_{s,k}$, for all $k = 1, \ldots, K$.

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Ongoing Work and Open Issues

Open issues:

1. Global results (NHRW); for the log-otptimal rule see
2. Aggregate risk, see
3. More general demands (CARA, ...)
4. General learning
5. Long lived assets (endowments)
6. Wealth-driven selection and stylized facts
   - excess volatility
   - excess covariance
   - equity premium puzzle
   - ...

Thank You!

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Centro di Ricerca Matematica Ennio De Giorgi

INSTITUTE OF ECONOMICS — Scuola Superiore Sant'Anna

Summer School of Mathematics for Economics and Social Sciences

16 - 20 September 2013

The **"Summer School of Mathematics for Economics and Social Sciences"** aims to improve the knowledge of mathematical methods among graduate students in economics and social sciences, with a focus on those techniques which albeit widespread in use are not properly covered in typical graduate programs. The School is an interdisciplinary venue intended to foster the interaction of people coming from the too often separated communities of mathematical and social scientists. It is organized by the *Mathematics Research Center "Ennio De Giorgi"* and supported by the *International Doctoral Program in Economics of the Scuola Superiore Sant'Anna*.

Dates: from 16 to 20 September2013

Venue: Conservatorio di Santa Chiara, San Miniato, Italy

Topics: Information theory, chaos and ergodicity with application to data analysis

Lecturer: Stefano Marmi, Scuola Normale Superiore, Pisa

Fabrizio Lillo, Scuola Normale Superiore, Pisa

Participation

The participation is subject to a selection. Only 20-25 positions are available. Financial support for board and accommodation will be provided.

On-line applications should be made at http://crm.sns.it/event/276/financial.html

School in San Miniato: `http://crm.sns.it/event/276/`

My own investigations

- G. Bottazzi and P. Dindo *Evolution and market behavior with endogenous investment rules* `http://www.lem.sssup.it/WPLem/2010-20.html`
- G. Bottazzi and P. Dindo, *Selection in asset markets: the good, the bad, and the unknown*, Journal of Evolutionary Economics

Deterministic model with noise:

- M. Anufriev, G. Bottazzi, M. Marsili and P. Pin *Excess Covariance and Dynamic Instability in a Multi-Asset Model*, Journal of Economic Dynamics and Control, 36(8), pp.1142-1161, 2012
- M. Anufriev, G. Bottazzi *Market Equilibria under Procedural rationality*, Journal of Mathematical Economics, 46(6), pp.1140-1172, 2010
- M.Anufriev, G.Bottazzi and F.Pancotto *Equilibria, Stability and Asymptotic Dominance in a Speculative Market with Heterogeneous Agents* Journal of Economic Dynamics and Control, 30, pp. 1787-1835, 2006