

The relevance of distributional properties in firm dynamics: theory and empirics

Giulio Bottazzi and Federico Tamagni

Institute of Economics
Scuola Superiore Sant'Anna

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FACTS

~~FACTS~~

REGULARITIES

IT DEPENDS

IT DEPENDS

Examples

Q. What's the distribution of yearly firm growth rates in a given period, of a given industry in a given country?

A. It's an exponential power distribution with exponent equal to 1, namely a “Laplace” distribution.

Q. What's the scaling relation between firm size and growth rates volatility?

A. It's a power law with scaling coefficient equal to 0.2

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Firm growth rate

Consider the dynamics of firms in a sector as different realization of the same (conditional) stochastic process.

Let g_i be the growth rate of firm i . The distribution of the g 's

$$F(x) = \text{Prob} \{g \leq x\}$$

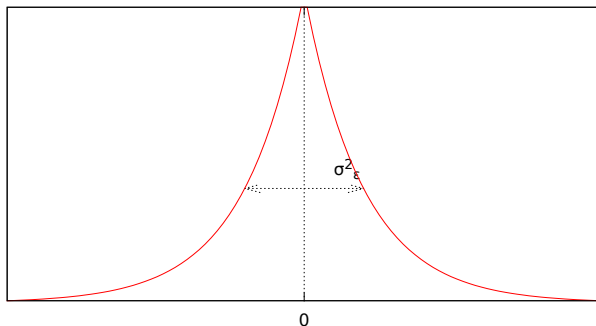
and the probability densities of the g 's

$$f(x) = \text{Prob} \{g \sim x\}$$

It's just a central limit theorem

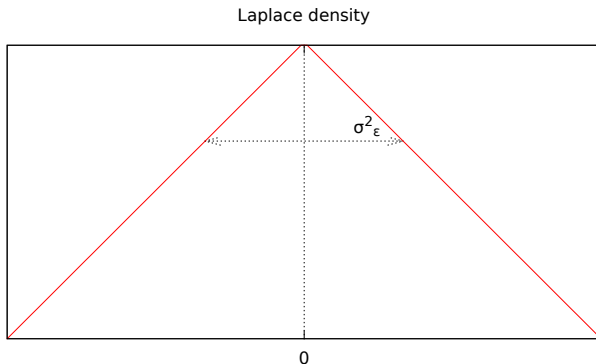
If the observed growth rates are the sum of a large and **random** number of zero mean random shocks then ...

Laplace density



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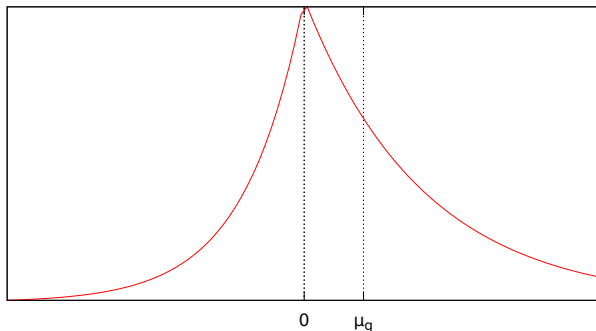
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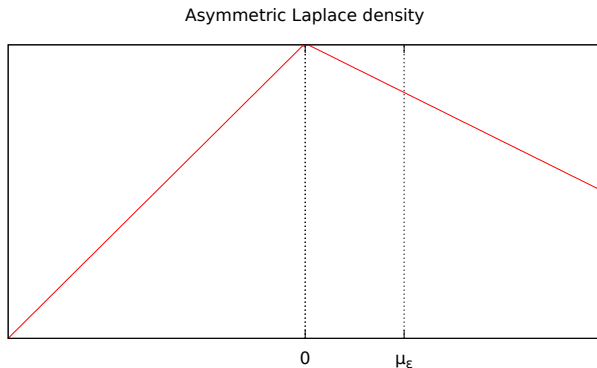
If the observed growth rates are the sum of a large and **random** number of non-zero mean random shocks then ...

Asymmetric Laplace density



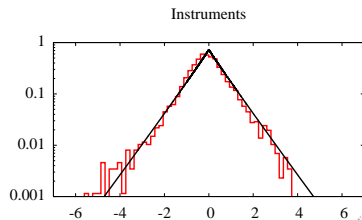
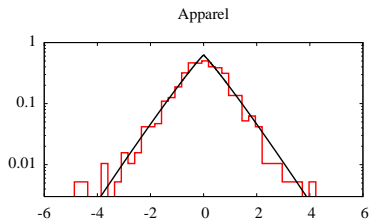
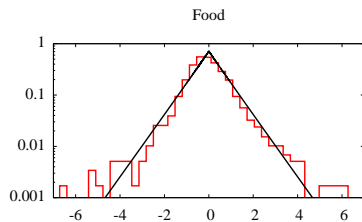
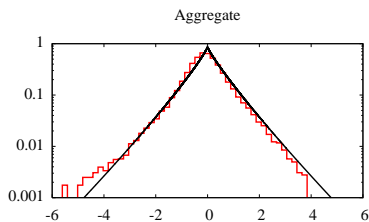
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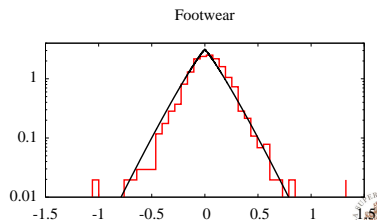
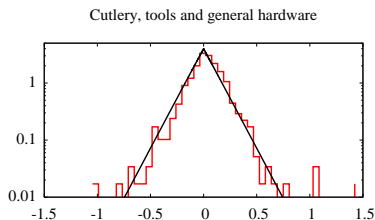
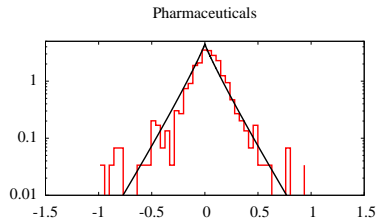
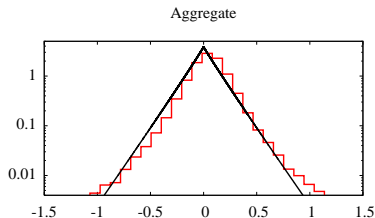
Empirical Growth Rates Densities - U.S.

COMPUSTAT. Two digits sectors. Some year from 1982 to 2001.



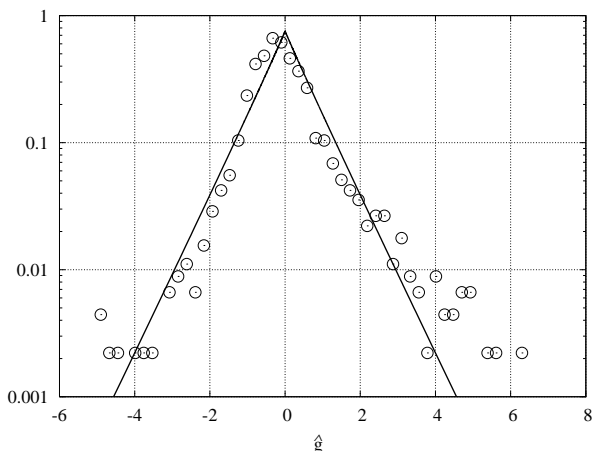
Empirical Growth Rates Densities - ITA

MICRO.1 by the Italian Statistical Office (ISTAT). Three digits sectors. Some year from 1989 to 1996.



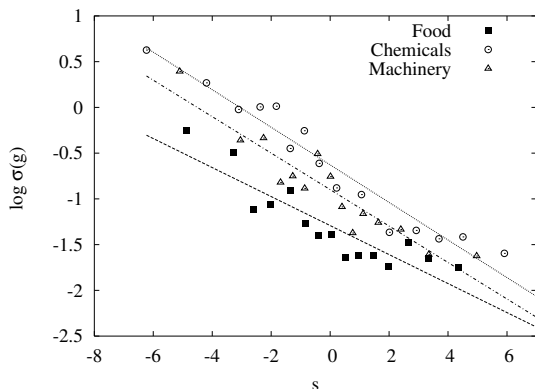
Empirical Growth Rates Densities - Pharma

PHID Top pharmaceutical firms in United States, United Kingdom, France, Germany, Spain, Italy and Canada for the Some year between 1987 and 1997.



Scaling of variance

Assuming path dependency and limited capability of exploiting business opportunities ...



0.2 is between 0, no “portfolio effect”, and 0.5, no limit to diversification.

Regularities are for what?

Rare as they are, they present invaluable opportunities for researchers:

1. They are interesting *per se* as they hint to general properties which should be investigated;
2. They provide extremely valuable testbed for the validation and calibration of our more general (and usually deductive) models;
3. They represents statistical tools allowing a better representation and a more precise measure, and thus a deeper undersading, of the observed phenomena

Focus on the third point, trying to disentangle the FC-growth nexus.

Available evidence on FCs and firms' dynamics

1. **structured / complex impact of FCs**: FCs affect many dimensions of firms' decisions and evolution
 - investment/divestment decisions
 - decision to expand production or entering new markets
 - cash management
 - R&D policies
2. Qualitative evidence on reaction to crises (Campello, Graham and Campbell, NBER2009) suggests **heterogenous impact of FC**:
 - “Pinioning” effect: firms facing good opportunities tend to bypass attractive investment projects
 - “Loss reinforcing” effect: firms facing poor growth opportunities display higher propensity to sell off productive assets to generate funds, further deteriorating growth prospects

Traditional approach: FC and central tendency

Augmented regression

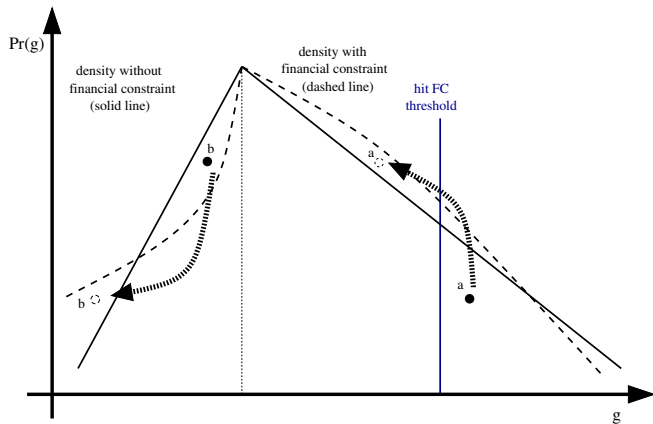
$$s_t - s_{t-1} = c + \lambda s_{t-1} + \beta \text{FC-Proxy} + \epsilon_t$$

Limitation: **it only captures central effect of FCs on growth**. Not able to describe the richness of the qualitative observations.

FCs and the distribution of growth rates

Qualitative evidence: “pinioning” and “loss reinforcing”

ASYMMETRIC DISTRIBUTIONAL EFFECT



going beyond

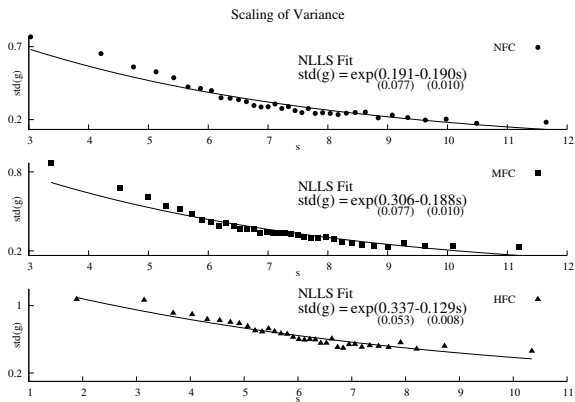
$$s_t - s_{t-1} = c_{FC} + \lambda_{FC} s_{t-1} + \sum_j \beta_j x_j + \sigma_{FC}(s_{t-1}) \epsilon_{FC,t} .$$

Several improvements:

- it is based on knowledge of regularities, heteroskedasticity is not a problem but rather a phenomenon;
- we are mostly interested in the distribution of residuals;

Result for $\sigma_{FC}(s)$

Often reported negative relation between the variance of growth $g_{i,t} = s_{i,t+1} - s_{i,t}$ and size.

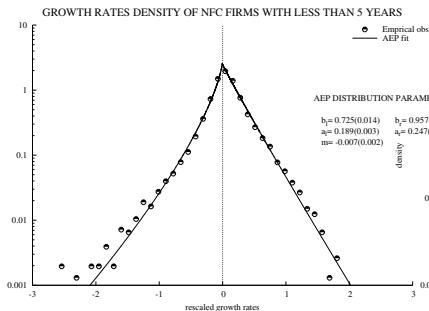


	<u>NFC</u>	Model 1	Model 2A	Model 2B
γ		-0.200*(0.001)	-0.194*(0.001)	-0.194*(0.001)
c		0.019*(0.001)	0.022*(0.001)	0.027*(0.002)
λ		-0.0001(0.0003)	-0.007*(0.001)	-0.008*(0.001)
$\ln(\text{Age}_{i,t})$			-0.021*(0.001)	-0.020*(0.001)
$\ln(\text{Assets}_{i,t-1})$			0.023*(0.001)	0.023*(0.001)
$\ln(\text{GOM}_{i,t-1})$			0.0002(0.001)	0.001(0.001)
	<u>MFC</u>			
γ		-0.204*(0.001)	-0.194*(0.001)	-0.195*(0.001)
c		-0.002(0.001)	0.0003(0.001)	-0.002(0.001)
λ		-0.0063*(0.0004)	-0.017*(0.001)	-0.017*(0.001)
$\ln(\text{Age}_{i,t})$			-0.037*(0.001)	-0.037*(0.001)
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	<u>HFC</u>			
γ		-0.164*(0.002)	-0.152*(0.003)	-0.151*(0.003)
c		0.006(0.003)	0.024*(0.003)	0.013*(0.004)
λ		-0.019*(0.002)	-0.046*(0.002)	-0.043*(0.002)
$\ln(\text{Age}_{i,t})$			-0.103*(0.003)	-0.104*(0.003)
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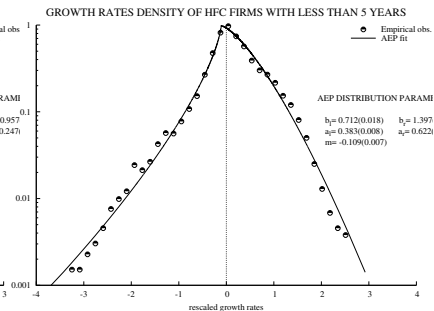
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Distribution of residuals by FC class: young firms

YOUNG NFC



YOUNG HFC

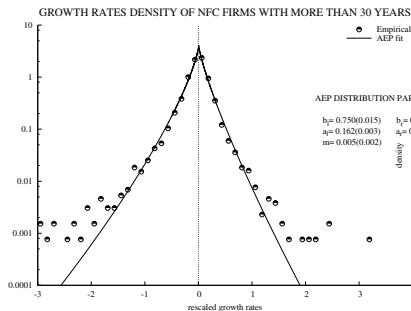


For younger firms, strong FCs :

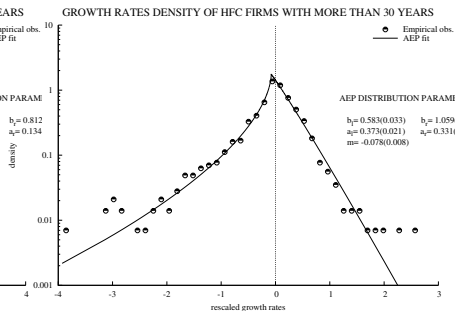
- slim down the right tail of the distribution, i.e. shift of probability mass from the tail to the central part of the distribution
- do not seem to have an effect on the left half

Distribution of residuals by FC class: old firms

OLD NFC



OLD HFC



For old firms, strong FCs:

- imply a very mild slim down of the right tail
- fatten up the left tail of the distribution, i.e. shift of probability mass from the central part to the tail of the distribution

Summing up

The effects of financial constraints on firms are manifold and impact on several aspects of growth dynamics, well beyond what can be captured by a shift in the expected growth rates.

The knowledge about regularities in firm dynamics allows to design a richer econometric model, which is better in identifying the phenomenon, at the same time providing a richer description of it.

Asymmetric Power Exponential distribution

$$f_{AEP}(x; \mathbf{p}) = \frac{1}{C} e^{-\left(\frac{1}{b_l} \left|\frac{x-m}{a_l}\right|^{b_l} \theta(m-x) + \frac{1}{b_r} \left|\frac{x-m}{a_r}\right|^{b_r} \theta(x-m)\right)}$$

where $\mathbf{p} = (b_l, b_r, a_l, a_r, m)$, $\theta(x)$ is the Heaviside theta function and C the normalization constant. **Back**

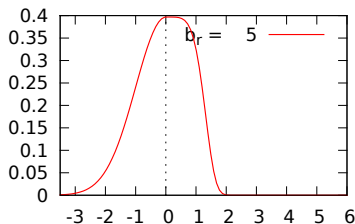
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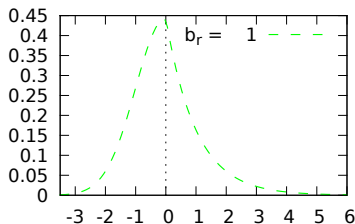


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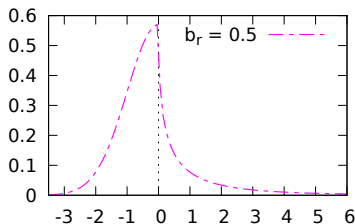


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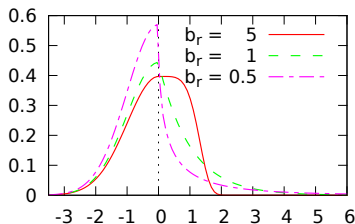


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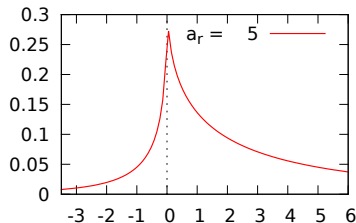
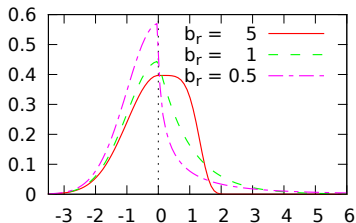
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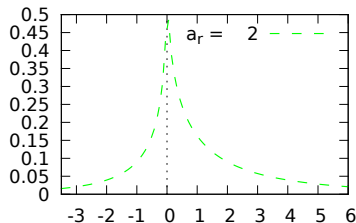
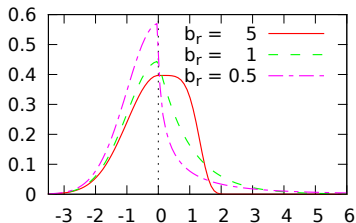
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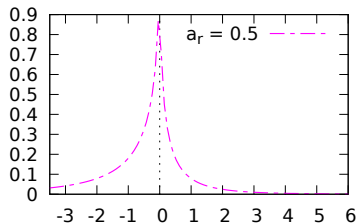
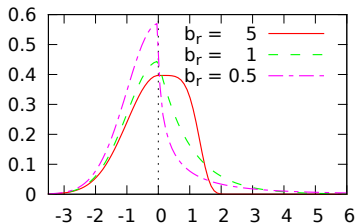
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